Higher Education and Oriental Studies (HEOS) – Vol 1(4): 64 – 77
www.heos.asia/ojs
e-ISSN 2785-9118
http://doi.org/10.54435/heos.v1i4.34

Differential Game Model for Disaster Relief Cooperation between Host Government and Private Sector

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ABSTRACT

The decision of the private sector to fulfil corporate social responsibility (CSR) has become an issue of common concern for the host country government and the private sector in collaborative disaster relief. From the perspective of social responsibility, this paper describes the different cooperation strategies of non-cooperation, cost subsidy, and collaborative cooperation, and constructs a differential game model of disaster relief cooperation between the government and the private sector. The rate of reputational decay has a significant effect on system benefits. The optimal CSR cost subsidy rate is related to marginal revenue and has nothing to do with other factors. Compared with the non-cooperative case, in the situation of the cost of subsidies, as the host country government and marginal revenue is greater than the private sector of the marginal revenue through government subsidies for the private sector in CSR and the government's optimal level of CSR efforts remained unchanged, the optimal level of CSR efforts to improve the private sector, the government, and the private sector's earnings are improving; Compared with the cost subsidy, the optimal CSR effort level of the government and the private sector increases in the case of collaborative cooperation, and so does the system benefit.

Keywords: disaster relief; differential game; corporate social responsibility; cost of subsidies

INTRODUCTION

The occurrence of natural disasters has hindered the development of colleges and universities to a certain extent. The occurrence of disasters not only brought uncertainty to colleges and universities, but also instability (He, 2021). When there is a large-scale earthquake disaster, universities will face unprecedented risks. In the 2010 Haiti earthquake, schools, public and commercial buildings are destroyed (Leaning et al., 2013), 7010 schools in 2021 were damaged by flood to varying degrees, thus the government should strengthen the construction of colleges and universities to minimize losses in the disaster, and the relationship between the university and government is an important issue of social and educational circles to discuss.

In the next 50 years, nature and man-made disasters are expected to increase fivefold (Thomas et al., 2005), the potential and destruction from disasters cannot be ignored, and considerable efforts must be made to mitigate their negative effects. The United Nations International Decade for Natural Disaster Reduction recognizes that the primary responsibility for disaster reduction lies with governments, but they also recognized the need for collaborative effort among governments, United Nations agencies, and other actors. Moreover, the Sendai Framework for Disaster Risk Reduction (SFDRR) proposed that the responsibility of disaster risk reduction and disaster risk management is upon all of society and all of its institutions. Host governments are political actors that exert significant influence on the activities of international humanitarian organizations (Dube et al., 2016). While some host governments have contributed to good performance by declaring disaster relief, the efficient path of government-mandated spending limits the sustainable development goals, complex natural disasters cannot be addressed by the government alone, and the private sector plays a key role in disaster response and post-disaster relief.

With the increasing severity of natural disasters, host governments and private sectors

are paying more attention to corporate social responsibility (CSR) investment, and CSR has become a differentiated strategic choice for enterprises to participate in market competition. During the Wenchuan earthquake in 2008, more than 100 enterprises donated more than 10 million yuan within a week. After the disaster, the extensive publicity of the media will provide full play to the influence of enterprises to fulfil their social responsibility, greatly improving the exposure rate of enterprises and bringing more profits to enterprises. However, the "Nike sweatshop" incident shows that the lack of CSR not only leads to the deterioration of cooperative relations among members but would also lead to the decline of enterprise competitiveness. India's Company Act of 2013 makes India one of the few countries with a mandatory corporate social responsibility system. Therefore, it is necessary to strengthen CSR management in disaster relief management.

Numerous scholars have studied CSR management in private sector enterprises. Private companies have professionals and are well-equipped to respond to disaster and recovery activities and encourage them to participate on the ground (Kanji et al., 2020). Kim found that CSR donations by corporate leaders may have a positive ripple effect on the multi-level reputation of enterprises and countries (Kim et al., 2021). Jia et al. (2015) and Bartkus et al. (2015) found that enterprises can improve their reputation through donations. Sanchez believes that enterprises would establish or maintain political relations with the government through charitable donations and win the favour and trust of the government (Carol, 2000). Disaster relief requires coordinated and cooperative effort between the government and the private sector. For example, Sadka (2007) discussed the public-private partnership from the perspective of public economics. Hausken et al. (2013) analysed the effect of disasters on the interaction strategies of governments and enterprises. In the disaster relief process, the private sector will weigh the costs and benefits of participating and choose whether to "share the pressure" with the government. To encourage the private sector to fulfil its social responsibilities, the host government usually adopts policies, such as tax incentives and financial subsidies. Meanwhile, the private sector can rely on its daily business operation to rotate the update (Wang et al., 2019), which helps in reducing the loss of inventory and reserve value from the perspective of the fixed shelf life of disaster relief materials (Zhang et al., 2019). It has been proven that coordination between different agencies can increase the effectiveness of institutions (Ergun et al., 2014; Horney et al., 2016; Zhang, 2021), and good interaction between the government and the private sector can improve the efficiency of disaster relief by reducing duplicate services and increasing resource utilization.

In recent years, scholars have paid attention to disaster relief cooperation between governments and private sectors. However, existing studies focused on the effect of corporate philanthropic activities on corporate image and corporate performance through research methods, and the lack of in-depth discussions on the game relationship between government and private sector activities, most of which may provide some policy suggestions. Game theory is used to analyse the interaction of multiple decision-makers in various situations (e.g., competition or cooperation), Coles et al. (2011) presented a view on how to incorporate game theory principles into the decision-making of disaster relief operations agencies. Zhuang et al. (2014) proposed a game theory model to investigate the interaction between non-profit disclosure and individual giving. Xu et al. (2016) exhibited a supply chain interruption model in a defender-attacker game that was established.

Therefore, this paper attempts to enrich the content of quantitative research in this field through mathematical modelling and describe the decision-making process between the government and the private sector in the process of disaster relief. In disaster relief, the government and the private sector are independent of bound rationality, in which a decision can be difficult to achieve in a stable equilibrium, and the differential game is dealing with all parties or more consecutive time conflicts, competition or a cooperation important dynamic

game model, which is widely used in dynamic competitive advertising (Liu et al., 2012; Guan et al., 2020), and corporate reputation management fields. Thus, the choice differential game has certain practicality and rationality. This paper studies the CSR effort level of the government and the private sector from the dynamic perspective, establishes the differential game model of the government and the private sector, analyses the dynamic optimization problem in the dynamic environment of non-cooperation, cost subsidy and collaborative cooperation, and uses the Hamilton-Jacobi-Bellman (HJB) equation. The optimal level of CSR effort, the optimal trajectory of CSR reputation, and the optimal value of revenue of the government and the private sector are calculated, while the key factors affecting the level of CSR effort of the government and the private sector are discussed to improve the efficiency of disaster relief and provide a theoretical basis for the actual disaster relief decisions of the private sector.

The structure of the article is as follows. Section 2 presents the model setup and symbol definition. Section 3 presents the differential model of the disaster relief cooperation between the government and private sector. Section 4 presents, discusses, and compares the feedback solution of the game. Section 5 describes a numerical analysis. The conclusions are presented in Section 6.

THE MODEL SETUP

This paper considers the disaster relief system, which comprises the host government and the private sector as the research object and studies the differential game between the host government and the private sector. There are three control variables, which are the government's effort level, the private sector's effort level, the proportion of government's cost subsidy to the private sector, and a state variable, CSR reputation. For the convenience of subsequent expressions, the main variables and parameters of the model and their meanings are provided, as shown in Table 1.

TABLE 1. Main parameters of the model and their meanings

Variables	Definitions	Parameters	Definitions
$E_G(t)$	Level of government effort, control variables	λ_G	The influence coefficient of government social responsibility input on the demand of disaster relief enterprises, $\lambda_G > 0$
$E_S(t)$	Level of private sector effort, control variables	λ_E	The influence coefficient of private sector social responsibility input on disaster relief enterprise demand, $\lambda_E > 0$
$S(t)$	Government subsidies to private sector costs, control variables $0 < S(t) < 1$	η_G	The effort cost coefficient of government to fulfill social responsibility
$G(t)$	CSR reputation, status variables	η_S	Cost coefficient of the private sector to fulfill social responsibility
Parameter	Definitions	θ	The influence coefficient of CSR reputation on the demand of disaster relief enterprises
α	The influence coefficient of the private sector's performance of social responsibility on CSR reputation	δ	Rate of reputational decay, $0 < \delta < 1$
β	The influence coefficient of government's fulfillment of social responsibility on CSR reputation	ρ	discount rate, $0 < \rho < 1$
π_G	Government marginal revenue, $\pi_G > 0$	π_S	Private sector marginal revenue, $\pi_S > 0$

In various disaster relief environments, the optimal effort level and CSR unit cost of the

private sector are different under various circumstances, but they basically conform to the general law that cost burden is positively correlated with effort input. Referring to the hypothesis of the effort cost of both parties in the game in literature (Guan et al., 2020), it can be concluded that:

The cost of the host government and the private sector to fulfil their social responsibilities is related to the effort level, while the effort cost of the host government and the private sector to fulfil their social responsibilities at time t is

$$C_G(t)=\frac{\eta_G E_G^2(t)}{2}\ ,C_S(t)=\frac{\eta_S E_S^2(t)}{2} \tag{1}$$

CSR reputation is related to the CSR effort level of the government and the private sector. The Nerlove-Arrow goodwill model is adopted to represent the change process of corporate reputation over time with reference to the idea of reference (Guan et al., 2020):

$$\dot{G}(t)=\alpha E_S(t)+\beta E_G(t)-\delta G(t),\ \ G(0)=G_0\geq 0 \tag{2}$$

The fulfilment of social responsibility by the government and the private sector has an important effect on the demand of disaster relief enterprises and its function is as follows:

$$Q(t)=\lambda_G E_G(t)+\lambda_S E_S(t)+\theta G(t) \tag{3}$$

Assuming that the government and the private sector have the same discount rate, both aimed to maximize their own returns in the infinite time zone, and their objective functions are as follows:

$$\begin{aligned}\max_{E_G} J_G&=\int_0^\infty e^{-\rho t}\left\{\pi_G\left[\lambda_G E_G(t)+\lambda_S E_S(t)+\theta G(t)\right]-\frac{1}{2}\eta_G E_G^2(t)\right\}dt\\ \max_{E_S} J_S&=\int_0^\infty e^{-\rho t}\left\{\pi_S\left[\lambda_G E_G(t)+\lambda_S E_S(t)+\theta G(t)\right]-\frac{1}{2}\eta_S E_S^2(t)\right\}dt\end{aligned} \tag{4}$$

EQUILIBRIUM ANALYSIS

SCENARIO N: NON-COOPERATIVE EQUILIBRIA

In the non-cooperative scenario, the government and the private sector aimed at the optimal level of their CSR effort, while their decision-making questions are as follows:

$$\begin{aligned}\max_{E_G} J_G&=\int_0^\infty e^{-\rho t}\left\{\pi_G\left[\lambda_G E_G+\lambda_S E_S+\theta G\right]-\frac{1}{2}\eta_G E_G^2\right\}dt\\ \max_{E_S} J_S&=\int_0^\infty e^{-\rho t}\left\{\pi_S\left[\lambda_G E_G+\lambda_S E_S+\theta G\right]-\frac{1}{2}\eta_S E_S^2\right\}dt\end{aligned}$$

In this case, the government and the private sector are independent and equal to each other, seeking for the optimal CSR effort level investment strategy to maximize their own benefits. Then, the feedback Nash equilibrium under the non-cooperative situation between the government and the private sector is obtained and the result is shown in Proposition 1.

Proposition 1: In the non-cooperative case, the equilibrium strategy (CSR effort level E^{N*}), the optimal trajectory of CSR reputation G^{N*} and the equilibrium result (objective

function V^{N*}) of the government and the private sector are as follows:

i) The balanced strategies of the government and the private sector are as follows:

$$E_G^{N*} = \frac{\pi_G}{\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right), E_S^{N*} = \frac{\pi_S}{\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right).$$

(ii) The optimal trajectory of CSR goodwill is $G^{N*}(t) = \left(G_0 - G_\infty^N\right)e^{-\delta t} + G_\infty^N$

where $G_\infty^N = \frac{\beta\pi_G}{\delta\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right) + \frac{\alpha\pi_S}{\delta\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right).$

(iii) The equilibrium results for the government and the private sector are as follows:

$$V_G^{N*} = \frac{\pi_G\theta}{\rho+\delta}G + \frac{\pi_G^2}{2\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{\pi_G\pi_S}{\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2,$$

$$V_S^{N*} = \frac{\pi_S\theta}{\rho+\delta}G + \frac{\pi_G\pi_S}{\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{\pi_S^2}{2\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2.$$

Proof: According to optimal control theory, continuous bounded differential function is constructed $V_i(G), i \in (G,S)$, for any $G \geq 0$, both satisfying the HJB equation.

$$\rho V_G^N = \max_{E_G}\left[\pi_G\left(\lambda_G E_G + \lambda_S E_S + \theta G\right) - \frac{1}{2}\eta_G E_G^2 + V_G^{N'}\left(\alpha E_S + \beta E_G - \delta G\right)\right] \quad (5)$$

$$\rho V_S^N = \max_{E_S}\left[\pi_S\left(\lambda_G E_G + \lambda_S E_S + \theta G\right) - \frac{1}{2}\eta_S E_S^2 + V_S^{N'}\left(\alpha E_S + \beta E_G - \delta G\right)\right] \quad (6)$$

Solve the first-order partial derivatives of E_G and E_S on the right end of Equation (5) and Equation (6), respectively, and make the partial derivatives equal to zero to obtain the following:

$$E_G = \frac{\lambda_G\pi_G + \beta V_G^{N'}}{\eta_G}, E_S = \frac{\lambda_E\pi_S + \alpha V_S^{N'}}{\eta_S} \quad (7)$$

After substituting Equation (7) into Equation (5) and Equation (6), it can be denoted that:

$$\rho V_G^N = \left(\pi_G\theta - \delta V_G^{N'}\right)G + \frac{\left(\lambda_S\pi_G + \alpha V_G^{N'}\right)\left(\lambda_S\pi_S + \alpha V_S^{N'}\right)}{\eta_S} + \frac{\left(\lambda_G\pi_G + \beta V_G^{N'}\right)^2}{2\eta_G} \quad (8)$$

$$\rho V_S^N = \left(\pi_S\theta - \delta V_S^{N'}\right)G + \frac{\left(\lambda_E\pi_S + \alpha V_S^{N'}\right)^2}{2\eta_S} + \frac{\left(\lambda_G\pi_S + \beta V_S^{N'}\right)\left(\lambda_G\pi_G + \beta V_G^{N'}\right)}{\eta_G} \quad (9)$$

According to the structural characteristics of differential equations (8) and (9), the linear optimal value function about G is the solution of the HJB equation. Let the specific expression of the optimal utility function of the government and the private sector be

$$V_G^N(G) = a_1 G + b_1, V_S^N(G) = a_2 G + b_2 \quad (10)$$

where a_1, b_1, a_2 and b_2 are constants. Substitute the derivative of Equation (10) with respect to G into Equation (8) and Equation (9), and obtain

$$a_1 = \frac{\pi_G \theta}{\rho + \delta}, b_1 = \frac{\pi_G^2}{2\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{\pi_G \pi_S}{\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2$$
$$a_2 = \frac{\pi_S \theta}{\rho + \delta}, b_2 = \frac{\pi_G \pi_S}{\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{\pi_S^2}{2\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2$$

Substitute a_1 and a_2 into Equation (7) to obtain the optimal CSR effort level of the government and the private sector. Substitute the optimal effort level into Equation (2) to solve the differential equation and obtain the optimal trajectory of CSR reputation. Substitute a_1, b_1, a_2, and b_2 into Equation (10) to obtain the optimal benefits of the government and the private sector.

As shown in Proposition 1, government and the private sector determined the optimal strategy from the perspective of maximizing their own benefits, without considering the overall benefits of the disaster relief system. The optimal CSR effort level of the private sector is negatively correlated with the effort cost coefficient. Therefore, the government bears part of the cost of the private sector's efforts, which can encourage the private sector to invest in the CSR effort level.

SCENARIO B: COST SUBSIDY EQUILIBRIA

Assuming that the government plays a leading role in the rescue, the government provides some CSR cost subsidies to the private sector to stimulate the private sector to invest more CSR effort. From the perspective of long-term dynamics, the two sides play a sequential non-cooperative game, and the decision order is as follows: First, the government determines its own CSR effort level and CSR cost subsidy rate to the private sector; Second, the private sector determines its own CSR effort level according to the government's decision, in which the decision-making questions of both parties are as follows:

$$\max_{E_G, S_1} J_G^B = \int_0^\infty e^{-\rho t}\left\{\pi_G\left[\lambda_G E_G(t) + \lambda_E E_E(t) + \theta G(t)\right] - \frac{1}{2}\eta_G E_G^2(t) - \frac{1}{2} S\eta_S E_S^2(t)\right\} dt$$
$$\max_{E_S} J_S^B = \int_0^\infty e^{-\rho t}\left\{\pi_S\left[\lambda_G E_G(t) + \lambda_S E_S(t) + \theta G(t)\right] - \frac{1}{2}(1-S)\eta_S E_S^2(t)\right\} dt$$

In this situation, on the level of CSR effort between the government and the private sector decisions that constitute a Stackelberg game using backward induction, solving the optimal control problem of the private sector is followed by solving the optimal control problem of the government, and under the condition that the government and the private sector cost subsidy in the feedback Nash equilibrium, solving the results are shown in proposition 2.

Proposition 2: In the case of cost subsidy, the equilibrium strategy of the government and private sector (CSR effort level E^{B*}, the government subsidy rate S^*), optimal trajectory of CSR reputation G^{B*}, and equilibrium result of the government and private sector (objective function V^{B*}) are as follows:

(i) The balancing strategies of the government and the private sector are

$$E_G^{B*} = \frac{\pi_G}{\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right), E_S^{B*} = \frac{2\pi_G + \pi_S}{2\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right), S^* = \begin{cases} \dfrac{2\pi_G - \pi_S}{2\pi_G + \pi_S}, & 2\pi_G > \pi_S \\ 0, & 2\pi_G \le \pi_S \end{cases}.$$

(ii) The optimal trajectory of CSR reputation is $G^{B*}(t) = \left(G_0 - G_\infty^B\right)e^{-\delta t} + G_\infty^B$

where $G_{\infty}^{B}=\frac{\beta\pi_G}{\delta\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right)+\frac{\alpha\left(2\pi_G+\pi_S\right)}{2\delta\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)$.

(iii) The optimal returns of the government and the private sector are as follows:

$$V_G^{B*}=\frac{\pi_G\theta}{\rho+\delta}G+\frac{\pi_G^2}{2\rho\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right)^2+\frac{\left(2\pi_G+\pi_S\right)^2}{8\rho\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)^2$$

$$V_S^{B*}=\frac{\pi_S\theta}{\rho+\delta}G+\frac{\pi_G\pi_S}{\rho\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right)^2+\frac{\pi_S\left(2\pi_G+\pi_S\right)}{4\rho\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)^2$$

Proof: According to optimal control theory, continuous bounded differential function is constructed $V_i(G), i\in(G,S)$. For any $G\geq 0$, both satisfied the HJB equation.

$$\rho V_S^B=\max_{E_S}\left[\pi_S\left(\lambda_G E_G+\lambda_S E_S+\theta G\right)-\frac{1}{2}(1-S)\eta_S E_S^2+V_S^{B'}\left(\alpha E_S+\beta E_G-\delta G\right)\right] \quad (11)$$

Take the first-order partial derivative of the function at the right end of Equation (11) with respect to E_S and make the partial derivative equal to zero, and obtain the following:

$$E_S^B=\frac{\lambda_S\pi_S+\alpha V_S^{B'}}{(1-S)\eta_S} \quad (12)$$

Similarly, the government optimal value function satisfies the HJB equation:

$$\rho V_G^B=\max_{E_G,S}\left[\pi_G\left(\lambda_G E_G+\lambda_S E_S+\theta G\right)-\frac{1}{2}\eta_G E_G^2-\frac{1}{2}S\eta_S E_S^2+V_G^{B'}\left(\alpha E_S+\beta E_G-\delta G\right)\right] \quad (13)$$

Substitute Equation (12) into Equation (13), then take the first partial derivative of the right end of (13) with respect to ES and S, and make the partial derivative equal to zero, then

$$E_G^B=\frac{\lambda_G\pi_G+\beta V_G^{B'}}{\eta_G} \quad (14)$$

$$S=\begin{cases}\dfrac{2\left(\lambda_S\pi_G+\alpha V_G^{B'}\right)-\left(\lambda_S\pi_S+\alpha V_S^{B'}\right)}{2\left(\lambda_S\pi_G+\alpha V_G^{B'}\right)+\left(\lambda_S\pi_S+\alpha V_S^{B'}\right)} & 2B>A\\ 0 & 2B\leq A\end{cases} \quad (15)$$

where $A=\lambda_S\pi_S+\alpha V_S^{B'}, B=\lambda_S\pi_G+\alpha V_G^{B'}$.

Substitute equations (12), (14), and (15) into Equations (13) and (11) to obtain

$$\rho V_G^B=\left(\pi_G\theta-\delta V_G^{B'}\right)G+\frac{\left(\lambda_G\pi_G+\beta V_G^{B'}\right)^2}{2\eta_G}+\frac{\left[2\left(\pi_G\lambda_E+\alpha V_G^{B'}\right)+\lambda_E\pi_S+\alpha V_S^{B'}\right]^2}{8\eta_S} \quad (16)$$

$$\rho V_S^B=\left(\pi_S\theta-\delta V_S^{B'}\right)G+\frac{\left(\lambda_E\pi_S+\alpha V_S^{B'}\right)\left[2\left(\lambda_E\pi_G+\alpha V_G^{B'}\right)+\left(\lambda_E\pi_S+\alpha V_S^{B'}\right)\right]}{4\eta_S}+\frac{\left(\lambda_G\pi_S+\beta V_S^{B'}\right)\left(\lambda_G\pi_G+\beta V_G^{B'}\right)}{\eta_G} \quad (17)$$

According to the structural characteristics of differential equations (16) and (17), the linear optimal value function of G is the solution of the HJB equation. Let the specific expression of the optimal utility function of the government and the private sector be

$$V_G^B(G) = a_4\tau + b_4, V_S^B(G) = a_5\tau + b_5 \tag{18}$$

where a_4, b_4, a_5, and b_5 are constants. Substitute the derivative of Equation (18) with respect to G into Equation (16) and equation (17), and obtain the following:

$$a_4 = \frac{\pi_G\theta}{\rho+\delta}, b_4 = \frac{\pi_G^2}{2\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{(2\pi_G+\pi_S)^2}{8\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2$$
$$a_5 = \frac{\pi_S\theta}{\rho+\delta}, b_5 = \frac{\pi_G\pi_S}{\rho\eta_G}\left(\lambda_G + \frac{\beta\theta}{\rho+\delta}\right)^2 + \frac{\pi_S(2\pi_G+\pi_S)}{4\rho\eta_S}\left(\lambda_S + \frac{\alpha\theta}{\rho+\delta}\right)^2$$

Substitute a_4 and a_5 into Equations (12), (14), and (15) to obtain the optimal CSR effort level of the government and private sector and the optimal CSR cost subsidy rate of the government to the private sector. Substitute the optimal CSR effort level into Equation (2) and solve the differential equation to obtain the optimal trajectory of CSR reputation. Substitute a_4, b_4, a_5, and b_5 into Equation (18) to obtain the optimal benefits of the government and private sector. Thus, Theorem 2 is proven.

In the case of cost subsidy, the host government shares the cost of CSR effort for the private sector only when the marginal benefit of the host government is greater than half of the marginal benefit of the private sector (that is, $2\pi_G > \pi_S$), and the optimal CSR cost subsidy rate is only related to the marginal benefit of both sides and has nothing to do with other parameters. The optimal effort level of the private sector is positively correlated with the subsidy, which indicates that the government and the private sector may still decide their own effort level by maximizing their own interests, but the government subsidy can stimulate the private sector to improve its effort level. Subsidies are positively correlated with the marginal revenue of the government and negatively correlated with the marginal revenue of the private sector. It shows that the higher the marginal revenue of the government, the larger the proportion of government subsidies to the private sector; the smaller the marginal revenue of the private sector, the larger the proportion of government subsidies to the private sector.

SCENARIO C: COOPERATIVE EQUILIBRIA

The government and the private sector reached binding agreements in advance to make decisions from a holistic perspective to maximize the benefits of the disaster relief system. In this case, the decision problem is

$$\max_{E_G,E_S} J_T^C = \int_0^\infty e^{-\rho t}\left\{(\pi_G+\pi_S)\left[\lambda_G E_G(t) + \lambda_S E_S(t) + \theta G(t)\right] - \frac{1}{2}\eta_G E_G^2(t) - \frac{1}{2}\eta_S E_S^2(t)\right\}dt$$

In this case, the government and the private sector made the system achieve the optimal return by determining the optimal CSR input. Similar to Proposition 1, the proof process is incomplete, and the feedback Nash equilibrium under the collaborative cooperation between the government and the private sector is obtained. See Proposition 3 for the solution results.

Proposition 3: In the case of collaborative cooperation, the equilibrium strategy (CSR effort level E^{C*}), the optimal trajectory of CSR reputation G^{C*}, and the equilibrium result (objective function V^{C*}) of the government and private sector are as follows:

(i) The balanced strategies of the government and private sector are as follows:

$$E_G^{C*}=\frac{(\pi_G+\pi_S)}{\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right),\quad E_S^{C*}=\frac{(\pi_G+\pi_S)}{\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)$$

(ii) The optimal trajectory of CSR reputation is $G^{C*}(t)=\left(G_0-G_\infty^C\right)e^{-\delta t}+G_\infty^C$

where $G_\infty^C=\frac{\beta(\pi_G+\pi_S)}{\delta\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right)+\frac{\alpha(\pi_G+\pi_S)}{\delta\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)$

(iii) The system equilibrium result is as follows

$$V_T^{C*}=\frac{\theta(\pi_G+\pi_S)}{\rho+\delta}G+\frac{(\pi_G+\pi_S)^2}{2\rho\eta_G}\left(\lambda_G+\frac{\beta\theta}{\rho+\delta}\right)^2+\frac{(\pi_G+\pi_S)^2}{2\rho\eta_S}\left(\lambda_S+\frac{\alpha\theta}{\rho+\delta}\right)^2$$

COMPARISON AND ANALYSIS

First, we compared the optimal level of CSR effort between the government and the private sector under different scenarios.

Corollary 1: (1) The optimal CSR effort level of the government is satisfied $E_G^{N*}=E_G^{B*}<E_G^{C*}$, and $2\pi_G>\pi_S$, in which the optimal CSR effort level of the private sector is satisfied $E_S^{N*}<E_S^{B*}<E_S^{C*}$, $\left(1-S^*\right)E_S^{B*}=E_S^{N*}$; (2) The optimal CSR effort level of the private sector is satisfied $E_S^{N*}=E_S^{B*}<E_S^{C*}$, and $2\pi_G>\pi_S$, the optimal CSR effort level of the government is satisfied $E_G^{N*}<E_G^{B*}<E_G^{C*}$.

Compared with the non-cooperative situation, the government's optimal CSR effort level changes under the cost subsidy situation. When $2\pi_G>\pi_S$, the optimal level of CSR effort of the private sector increased because the government shared CSR costs for the private sector, which increased the enthusiasm of the private sector to undertake social responsibility. Furthermore, in the case of collaborative cooperation, the optimal CSR effort level of the government and the optimal CSR effort level of the private sector also increases because the CSR efforts of the government and the private sector are complementary to each other and would jointly affect the decision-making of both sides.

Second, we compared the CSR reputation in different situations.

Corollary 2: If $2\pi_G>\pi_S$, the optimal trajectory of CSR goodwill is satisfied $G^{N*}(t)<G^{B*}(t)<G^{C*}(t)$.

Compared with the non-cooperative situation, in the cost subsidy situation, because the optimal CSR effort level of the government remains unchanged while the optimal CSR effort level of the private sector increases, the CSR reputation and its stable value both increases. Furthermore, compared with the collaborative situation, the optimal CSR effort level of the government and the private sector is improved, in which CSR reputation and its stable value are also improved.

Finally, we compared the optimal value of government, private sector, and the whole under different circumstances.

Corollary 3: The optimal value of government income is satisfied $V_G^{N*}<V_G^{B*}$, and $2\pi_G>\pi_S$ the optimal value of private sector income is satisfied $V_S^{N*}<V_S^{B*}$.

Compared with the non-cooperative situation, the higher value of the government's income increases because the improvement of the private sector's best effort is affecting the reputation of the CSR. When $2\pi_G>\pi_S$, the government's CSR cost subsidies can improve the private sector's commitment to social responsibility, the benefit of the private sector revenue

increases.

Corollary 4: If $2\pi_G > \pi_S$, then the optimal value of system income satisfies $V^{N*} < V^{B*} < V^{C*}$.

Compared with the non-cooperative situation, in the cost subsidy situation, when the marginal revenue of the government is greater than half of the marginal revenue of the private sector, the optimal CSR effort level of the private sector increases, wherein the system income increases, although the optimal CSR effort level of the government remains unchanged. Compared with the cost subsidy situation, the optimal CSR effort level of the government and the private sector in the collaborative cooperation situation increases, while the system income increases, that is, the collaborative cooperation is the Pareto optimal scheme.

NUMERICAL ANALYSIS

In the differential game model of the performance of social responsibility by the host country government and the private sector, the reputation change and optimal return function have strong parameter dependence, and the comparative analysis complexity is high. To display the details of decision results in this study directly, a numerical analysis method is used to analyse the key indicators in different situations. According to the selection of numerical analysis data in literature (Liu et al., 2012) and combined with the actual situation, the following parameters are assigned for analysis: $\alpha = 2$, β=1, $\rho = 0.2$, $\lambda_G = 2$, $\lambda_S = 1$, $\theta = 0.5$, $\pi_G = 1$, $\pi_S = 1.5$, $\eta_G = 5$, $\eta_S = 7$, $G_0 = 10$.

FIGURE 1. CSR with high reputation attenuation

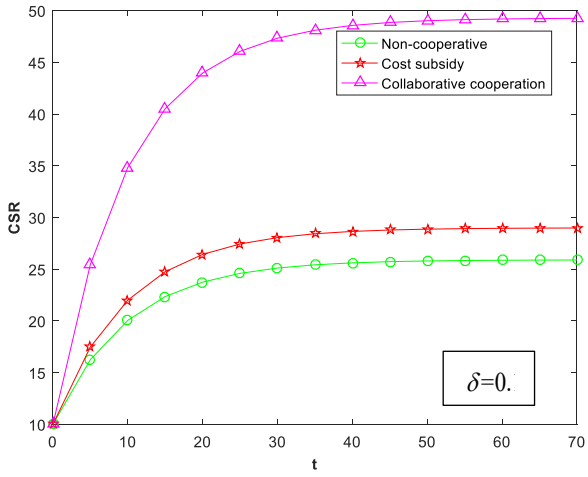
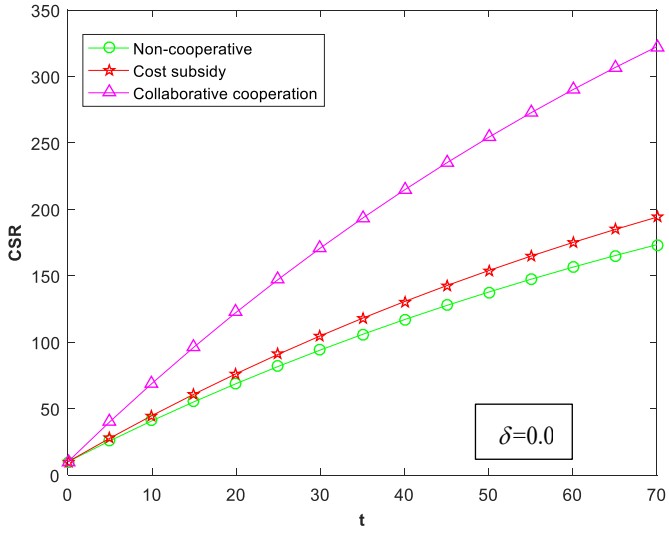


FIGURE 2. CSR with low reputation attenuation



As shown in Figure 1-2, cost subsidy can significantly improve CSR level, and the Pareto has a significant improvement effect in the long run. The cost subsidy provided by the host government to the private sector stimulates the CSR effort of the private sector, thus improving the CSR reputation of the private sector. The comparison shows that the level of CSR is significantly correlated with the rate of reputation decay. The faster the reputational decay rate is, the faster the CSR level increases in a short period. At the same time, the private sector must recognize the social complexity of the host country, as the host country strategy choice may not fulfil the Pareto improvement of economic benefits.

FIGURE 3. Returns under high reputation decay

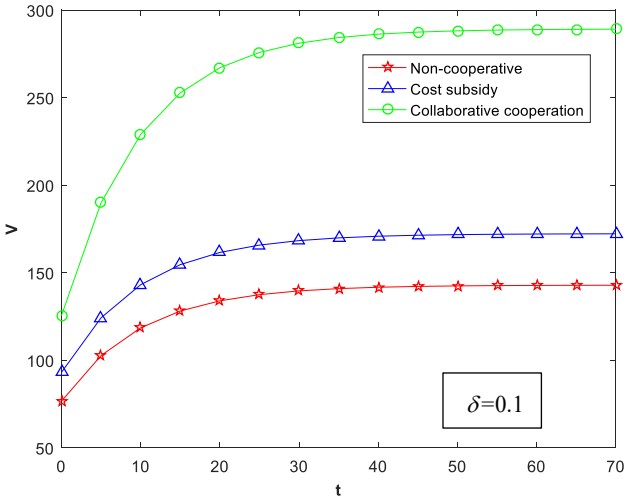
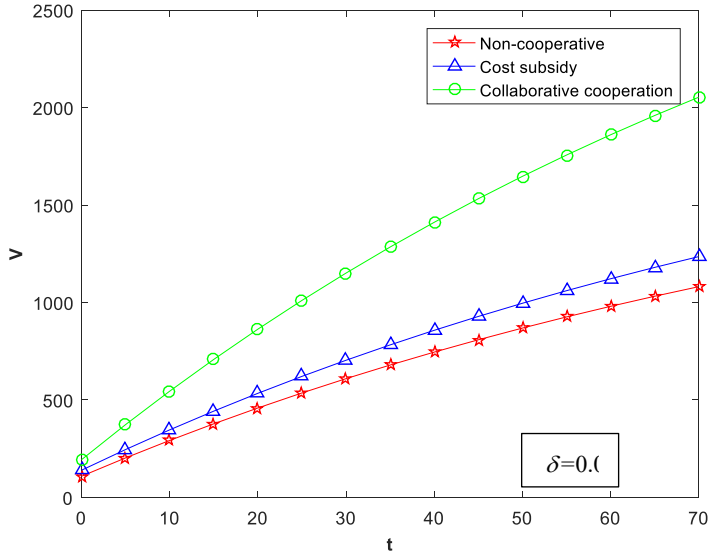


FIGURE 4. Returns from low reputation decay



As can be seen from Figure 3 and 4, in the case of high reputation attenuation, collaboration can obtain the highest returns in the long period. The cost subsidy is better in the whole cycle range, and the advantage is more significant in the long cycle, while the non-cooperation is worse in the whole cycle range. In the case of low reputation attenuation, the cooperative situation has an absolute advantage in the long period, while the non-cooperative situation has a poor performance in the whole period.

CONCLUSION

The results of this study can help the government in increasing the construction of colleges and universities, adjust the development direction of colleges and universities in the future, and strengthen the cultivation of application-oriented and versatile talents. Under the impetus of all conditions, the development of colleges and universities can adapt to changes and develop in time, which has important guiding significance.

In the process of major disaster relief, the participation of the private sector can relieve the pressure of the host government to a large extent. This paper divided the cooperation orientation of the host government to the private sector into three situations: non-cooperation, cost subsidy, and collaborative cooperation, thus constructing a differential game model from the perspective of CSR. The optimal CSR effort level, CSR reputation trajectory, and benefits of the host government and private sector are analysed, while the correctness of theoretical derivation is verified by numerical simulation.

Results show that CSR reputation and optimal returns are significantly correlated with the speed of reputation decay. When the speed of reputation decay is small, the CSR reputation can be achieved with a small investment level in the case of collaborative cooperation. When the decay rate of CSR reputation is large, the growth rate of CSR reputation is slow. Under the condition of high reputation attenuation, synergistic cooperation is the best in the whole cycle, while cost subsidy is the second best. (1) In the three game scenarios, the optimal CSR effort level of the government and private sector is directly proportional to the effect of CSR effort level on market demand, the marginal benefit of CSR effort level on CSR reputation, the effect coefficient of CSR reputation on market demand, and inversely proportional to the cost coefficient of CSR effort and reputation decay rate. Therefore, to improve the level of CSR effort, the private sector should take effective measures to reduce the cost of CSR management, slow down the rate of reputational decay, and expand the effect of CSR by increasing publicity

and other means. (2) In the case of cost subsidy, the optimal CSR cost subsidy rate is only related to marginal revenue. When $2\pi_G > \pi_S$, the government subsidizes the private sector. Marginal revenue is the key for the government to decide whether to subsidize the cost of CSR. (3) According to different cooperation modes, the government and the private sector should formulate their CSR effort in combination with their own needs and market changes to achieve the Pareto improvement of their own benefits and system benefits.

In practice, the effect of CSR investment often has a lag. In the future, the lag effect can be introduced into the study of CSR decision-making, and the system benefits can be reasonably distributed in the case of collaborative cooperation to achieve the Pareto optimization of individual benefits between the government and private sector.

REFERENCES

Bartkus, B. R., & Morris, S. A. (2015). Look Who's Talking: Corporate Philanthropy and Firm Disclosure. *Int. J. Bus. Soc. Res., 5*(1), 1-14.

Carol, M. (2000). Sanchez Motives for Corporate Philanthropy in El Salvador: Altruism and Political Legitimacy. *J. Bus. Ethics, 27*(4), 363-375.

Coles, J., & Zhuang, J. (2011). Decisions in disaster recovery operations: a game theoretic perspective on organization cooperation. *Journal of Homeland Security and Emergency Management*, *8*(1), 1-16.

Dube, N., Van, D., & Teunter, R. H. (2016). Host government impact on the logistics performance of international humanitarian organisations. *J. Oper. Manag., 47*, 44-57.

Ergun, Ö., Gui, L., Heier Stamm, J. L., Keskinocak, P., & Swann, J. (2014). Improving humanitarian operations through technology-enabled collaboration. *Production and Operations Management*, *23*(6), 1002-1014.

Guan, Z., Ye, T., & Yin, R. (2020). Channel coordination under Nash bargaining fairness concerns in differential games of goodwill accumulation. *European Journal of Operational Research*, *285*(3), 916-930.

Hausken, K., & Zhuang, J. (2013). The impact of disaster on the strategic interaction between company and government. *Eur. J. Oper Res., 225*(2), 363–376.

He, Z. (2021). Private colleges and universities under government management and the direction of the development of private colleges and universities-perspective of Guangzhou Xinhua university during the epidemic. *HEOS, 1*(3), 39-48.

Horney, J., Nguyen, M., Salvesen, D., Tomasco, O., & Berke, P. (2016). Engaging the public in planning for disaster recovery. *International journal of disaster risk reduction*, *17*, 33-37.

Jia, M., & Zhang, Z. (2015). News Visibility and Corporate Philanthropic Response: Evidence from Privately Owned Chinese Firms Following the Wenchuan Earthquake. *J. Bus. Ethics, 129*(1), 93-114.

Kanji, R., & Agrawal, R. (2020). Exploring the use of corporate social responsibility in building disaster resilience through sustainable development in India: An interpretive structural modelling approach. *Prog. Disaster Sci., 6*, 1-11.

Kim, S., & Ji, Y. R. (2021). Positive ripple effects of corporate leaders' CSR donations amid COVID-19 on corporate and country reputations: Multi-level reputational benefits of CSR focusing on Bill Gates and Jack Ma. *Public Relat. Rev. 47*, 1-11.

Leaning, J., & Guha-Sapir, D. (2013). Natural disasters, armed conflict, and public health. *N. Engl. J Med., 369*(19),1836–1842.

Liu, D., Kumar, S., & Mookerjee, V. S. (2012). Advertising strategies in electronic retailing: A differential games approach. *Information Systems Research*, *23*(3), 903-917.

Sadka, E. (2007). Public-Private Partnerships: A Public Economics Perspective. *Cesifo Econ.*

Stud, 3, 466-490.
Thomas, A. S., & Kopczak, L. (2005). R from logistics to supply chain management: The path forward in the humanitarian sector. *Fritz Institute, 15*, 1–15.
Wang, X., Fan, Y., & Liang, L. (2019). Augmenting fixed framework agreements in humanitarian logistics with a bonus contract. *Prod. Oper. Manag., 28*(8), 1921–1938.
Xu, J., Zhuang, J., & Liu, Z. (2016). Modeling and mitigating the effects of supply chain disruption in a defender–attacker game. *Annals of Operations Research*, *236*(1), 255-270.
Zhang, L., Tian, J., & Fung, R. (2019). Materials procurement and reserves policies for humanitarian logistics with recycling and replenishment mechanisms. *Comput. Ind. Eng., 127,* 709–721.
Zhang, L. (2021). Emergency supplies reserve allocation within government-private cooperation: A study from capacity and response perspectives. *Computers & Industrial Engineering*, *154*, 1-10.
Zhuang, J., Saxton, G. D., & Wu, H. (2014). Publicity vs. impact in nonprofit disclosures and donor preferences: A sequential game with one nonprofit organization and N donors. *Annals of Operations Research*, *221*(1), 469-491.

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